PROJECTILE MOTION

A simple way to determine the max height is to consider how far an object would need to drop to attain the speed V_y. (see red arrow)

Use $(V_f)^2 = (V_0)^2 + 2ad$ … with V_0 = zero.

Solve for d to get $d = (V_f)^2/2g$

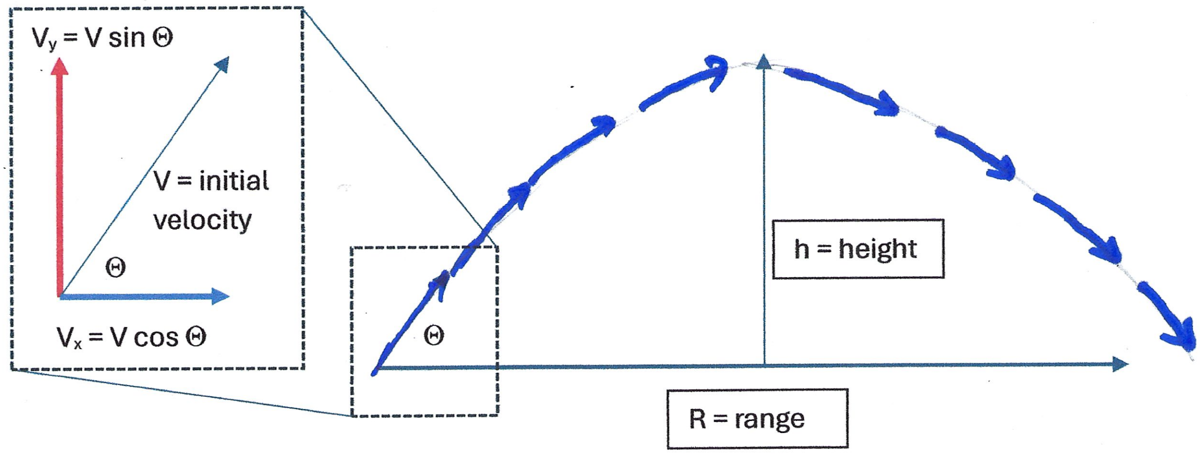
$$h = [V \sin(\Theta)]^2/2g$$

Motion Equations

$V_{ave} = (V_0 + V_f)/2$ $\quad d = (V_{ave})(t)$

$V_f = V_0 + at$ $\quad d = V_0 t + \frac{1}{2} a t^2$

$(V_f)^2 = (V_0)^2 + 2ad$



Time of flight

Since the path of the projectile is symmetric, it will spend as much time going up as coming down.

So, we only need to calculate the time for an object to fall (from the max height) till it reaches the "Y" component of velocity, that is, V_y. (See red arrow above)

$V_y = (a)(t)$ … where a is gravity.

Hence, $t = V_y/g$ or $= V_0 \sin(\Theta)/g$

But the TOTAL time is twice this:

$$T = 2 V \sin(\Theta)/g$$

Range

To calculate the range, multiply the total flight time by the horizontal initial velocity.

$R = (V_x)(T) = [V \cos \Theta] [2 V \sin(\Theta)/g]$

Combine terms

$$R = \frac{(V)^2 [(2 \sin \Theta)(\cos(\Theta)]}{g}$$

But from our double angle formulas.

$(2 \sin \Theta)(\cos(\Theta) = \sin(2\Theta)$

Hence:

$$R = (V^2/g) \sin 2\Theta$$